#### A.P. State Council of Higher Education Semester-wise Revised Syllabus under CBCS, 2020-21

Course Code:

Max Marks: 100

(15h)

(15h)

(15h)

## Four-year B.A. /B.Sc. (Hons) Domain Subject: **MATHEMATICS** IV Year B.A./B.Sc.(Hons)– Semester – V

#### **Course-7C: Number theory** (Skill Enhancement Course (Elective), 5 credits)

## 1. Learning Outcomes:

Students after successful completion of the course will be able to

- 1. Find quotients and remainders from integer division, study divisibility properties of integers and the distribution of primes.
- **2.** Understand Dirichlet multiplication which helps to clarify interrelationship between various arithmetical functions.
- 3. Comprehend the behaviour of some arithmetical functions for large n.
- 4. Understand the concepts of congruencies, residue classes and complete residues systems.
- 5. Comprehend the concept of quadratic residues mod p and quadratic non residues mod p.
- I. **Syllabus:** (Hours: Teaching:75 (incl. unit tests etc.05), Training:15)

## Unit – 1: Divisibility

- 1. Introduction, Divisibility, Greatest Common Divisor.
- 2. Prime numbers, The fundamental theorem of arithmetic, The series of reciprocals of the primes.
- 3. The Euclidean algorithm, The greatest common divisor of more than two numbers.

# Unit – 2: Arithmetical Functions and Dirichlet Multiplication

- 1. Introduction, The Mobius function $\mu(n)$ , The Euler totient function $\phi(n)$ , A relation connecting  $\phi$  and  $\mu$ , A product formula for  $\phi(n)$ .
- 2. The Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, The Mangoldt function  $\Lambda(n)$ .
- 3. Multiplicative functions, Multiplicative functions and Dirichlet multiplication, The inverse of a completely multiplicative function, Liouville's function  $\lambda(n)$ , The divisor functions  $\sigma_{\alpha}(n)$ .

# Unit – 3: Averages of Arithmetical Functions

- 1. Introduction, The big oh notation. Asymptotic equality of functions, Euler's summation formula, some elementary asymptotic formulas.
- 2. The average order of d(n), The average order of the divisor functions  $\sigma_{\alpha}(n)$ , The average order of  $\phi(n)$ .
- 3. The average order of  $\mu(n)$  and  $\Lambda(n)$ , The partial sum of a Dirichlet product, Applications of  $\mu(n)$  and  $\Lambda(n)$ .

## **Unit – 4: Congruences**

(15h)

- 1. Definition and basic properties of congruences, Residue classes and complete residue systems.
- 2. Linear congruences, reduced residue systems and the Euler-Fermat theorem. Polynomial congruences modulo p. Lagrange's theorem.
- 3. Applications of Lagrange's theorem, Simultaneous linear congruences. The Chinese remainder theorem. Applications of the Chinese remainder theorem.

## Unit – 5: Quadratic Residues and the Quadratic Reciprocity Law

- **1.** Quadratic Residues, Legendre's symbol and its properties, Evaluation of (-1/p) and (2/p), Gauss lemma,
- 2. The Quadratic reciprocity law, Applications of the reciprocity law, The Jacobi Symbol.
- **3.** Gauss sums and the quadratic reciprocity law, the reciprocity law for quadratic Gauss sums. Another proof of the quadratic reciprocity law.

## **III. Reference Books:**

- 1. Tom M.Apostol, Introduction to Analytic Number theory, Springer International Student Edition.
- 2. David, M. Burton, Elementary Number Theory, 2<sup>nd</sup> Edition UBS Publishers.
- 3. Hardy & Wright, Number Theory, Oxford Univ, Press.
- 4. Dence, J. B & Dence T.P, Elements of the Theory of Numbers, Academic Press.
- 5. Niven, Zuckerman & Montgomery, Introduction to the Theory of Numbers.
- 6. Web resources suggested by the teacher and college librarian including reading material.